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Sio-Iong Ao
Len Gelman *Editors*

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Preface

A large international conference on Advances in Engineering Technologies and Physical Science was held in London, UK, July 7–9, 2021, under the World Congress on Engineering 2021 (WCE 2021). The WCE 2021 is organized by the International Association of Engineers (IAENG); the Congress details are available at: http://www.iaeng.org/WCE2021. IAENG is a non-profit international association for engineers and computer scientists, which was founded originally in 1968. The World Congress on Engineering serves as good platforms for the engineering community to meet with each other and to exchange ideas. The conferences have also struck a balance between theoretical and application development. The conference committees have been formed with over three hundred committee members who are mainly research center heads, faculty deans, department heads, professors, and research scientists from over 30 countries. The congress is truly global international event with a high level of participation from many countries. The response to the Congress has been excellent. There have been more than four hundred manuscript submissions for the WCE 2021. All submitted papers have gone through the peer-review process, and the overall acceptance rate is 50.73%.

This volume contains eleven revised and extended research articles written by prominent researchers participating in the conference. Topics covered include mechanical engineering, engineering mathematics, computer science, electrical engineering, and industrial applications. The book offers the state of the art of tremendous advances in engineering technologies and physical science and applications and also serves as an excellent reference work for researchers and graduate students working on engineering technologies and physical science and applications.

Hong Kong — Prof. Sio-Iong Ao
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Piecewise Monotonic Fitting for Covid-19 Data Analysis of the United Kingdom During 31-01-2020 to 19-11-2021

Evangelos E. Vassiliou, Ioannis N. Perdikas, Demetrius E. Davos, and Ioannis C. Demetriou

Abstract The least squares piecewise monotonic data approximation method is applied to daily Covid-19 new cases and deaths data of the UK for the period 31-01-2020 to 19-11-2021. The data demonstrate wide variation in parts and noticeable peaks over time. We are interested in estimating turning points of the data in that the fit is useful to analyzing the progress of the pandemic. An enormous number of combinations of turning points need be considered in order to find an optimal combination, but the method provides quite efficiently a global solution. Our results show the efficacy of the piecewise monotonicity method in locating optimal turning points that are significant to the Covid-19 analyses. We consider the facts that influence the choice of the number of peaks. Our analysis provided us with insights regarding the driving forces behind the turning points that the method detected, which further may be helpful to management, as part of the information on which decisions will be made.

Keywords Approximation · Combinatorial problem · Covid-19 pandemic data · Divided difference of first order · Least squares fit · Peak · Piecewise monotonic · Turning point · United Kingdom

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1 Introduction

There has been an explosion of scientific publications on the Covid-19 pandemic (see, e.g., [7]). The application and analysis of the piecewise monotonic method to peak estimation of Covid-19 data, which we address here, are conducted for the first time. Daily Covid-19 new cases and deaths data of the UK for the period 31-01-2020 to 19-11-2021 demonstrate wide variation in time, as well as some noticeable peaks of different magnitude along the time range. We are interested in estimating peaks and trends of these data that can assist in the decision making policy. Since the underlying laws are unknown, the estimation problem is made rather difficult. We assume, though, that if the underlying function is piecewise monotonic, then the number of its turning points is substantially lower than the number of turning points of the data.

Therefore, we apply the method of Demetriou and Powell [6] that makes least changes to the data by imposing a limit on the number of sign changes in the sequence of the first differences. Specifically, if $k-1$ is the limit, then the approximated values consist of at most k monotonic sections. Thus, if some value of k can be derived from the form of the underlying function, or if the user is willing to try several values of k, then no other knowledge of this function is required. In this paper, we define "least changes" to the data by minimizing the sum of squares of changes subject to the smoothness condition. Further, two consecutive monotonic sections meet at a turning point. The positions of the turning points are also integer variables of the optimization calculation whose optimal values have to be found automatically. However, about $\mathcal{O}(n^{k-1})$ combinations of positions can occur, which makes it impossible to test each and every one separately. Demetriou and Powell [6] have studied this problem, and, furthermore, Demetriou [1, 2, 4] has developed algorithms and software that obtain a global solution in only $\mathcal{O}(n^2 + kn\log_2 n)$ computer operations. When $k = 1$ (monotonic case) or $k = 2$ (unimodal case), this work is reduced to $\mathcal{O}(n)$. This excellent efficiency is due to a decomposition property of the solution. Indeed, the method partitions the data into at most k disjoint sets of adjacent values and solves a $k = 1$ problem for each set.

Besides reduction of complexity when finding the optimal turning points, piecewise monotonic approximation method has some more advantages compared to other approximation techniques. First, it avoids the assumption that $f(x)$ has a parametric form. Second, the approximation process is a projection because, if it is applied to the approximated values, then no changes are made thereto. Third, the method of [6] is so fast in practice that it allows running for a sequence of integers k if a suitable value is not known in advance. Fourth, due to the nature of the constraints on the approximation components, any irregular errors in the data do not cause ripples to the piecewise monotonic fit. Moreover, any peak in the data does not introduce perturbations away from the peak.

The piecewise monotonic approximation method is outlined in Sect. 2. In Sect. 3, the method is applied to both Covid-19 new cases and deaths data of the UK, some technical aspects of the fit are provided, the estimation of peaks is illustrated, the results of the calculation are explained, and the effectiveness of the method is demonstrated. In Sect. 4, the results of Sect. 3 are used in further analyzing the progress of the Covid-19 pandemic in the UK, in conjunction with various important dates and events pertaining to the timeline of the pandemic. In Sect. 5, some conclusions are presented and the possibility of expanding this research is discussed.

All the experiments were handled routinely by the Fortran program of Demetriou [2], which implements a variant of the method outlined in Sect. 2. We provided just the data sequences and the value of k; the optimal turning points and the smoothed values were automatically delivered by the method.

Piecewise monotonic data approximation may be applied to a variety of situations in which peak estimations are required, but sufficient information is lacking to state a parametric form for the underlying function. Such situations are quite common, so applications of piecewise monotonic approximation can occur in several fields. Examples arise from time series [8], from spectroscopy [3, 5] and from curve fitting to physical data, to mention few. More applications appear in the reference list of [4].

2 Piecewise Monotonic Approximation

Let $\{(x_i, \phi_i) : i = 1, 2, \ldots, n\}$ be pairs of real numbers, where the abscissae $\{x_i : i = 1, 2, \ldots, n\}$ are in the strictly ascending order $x_1 < x_2 < \cdots < x_n$, and where ϕ_i is the measurement of some unknown underlying function, or a process, $f(x)$ at x_i. Due to errors of measurement, it is possible that the number of sign changes in the sequence $\{\phi_{i+1} - \phi_i : i = 1, 2, \ldots, n-1\}$ is much greater than the number in the sequence $\{f(x_{i+1}) - f(x_i) : i = 1, 2, \ldots, n-1\}$. The piecewise monotonic approximation method makes the least change to the data so that the new values $\{y_i : i = 1, 2, \ldots, n\}$ have at most $k-1$ sign changes in the sequence $\{y_{i+1} - y_i : i = 1, 2, \ldots, n-1\}$, where k is any positive integer. In this paper we define the "least" change to the data that gives the smoothing condition by choosing the L_2 norm in $\mathbb{R}^n$. Therefore, the piecewise monotonic method calculates numbers $\{y_i : i = 1, 2, \ldots, n\}$ that minimize the expression

$$\Phi(y_1, y_2, \ldots, y_n) = \sum_{i=1}^{n} (y_i - \phi_i)^2, \tag{1}$$

subject to the constraints

$$\left.\begin{array}{l} y_{t_{j-1}} \leq y_{t_{j-1}+1} \leq \cdots \leq y_{t_j}, \ \ j \text{ is odd} \\ y_{t_{j-1}} \geq y_{t_{j-1}+1} \geq \cdots \geq y_{t_j}, \ \ j \text{ is even} \end{array}\right\}, \tag{2}$$

where $\{t_j \colon j = 1, 2, \ldots, k-1\}$ are integers that satisfy the conditions

$$1 = t_0 \leq t_1 \leq \cdots \leq t_k = n. \tag{3}$$

The integers $\{t_j\colon j = 1, 2, \ldots, k-1\}$ are also variables of the optimization calculation. This is a formidable combinatorial optimization problem that requires $\mathcal{O}(n^{k-1})$ combinations of integers in order to find an optimal combination. However, Demetriou and Powell [6] have developed a highly efficient method that generates the solution in quadratic complexity with respect to n. The efficiency of the calculation of Demetriou and Powell takes advantage of two main properties of the best fit, but here we give no further details.

The first property is that the turning points of an optimal fit satisfy the interpolation conditions

$$y_{t_j} = \phi_{t_j}, \quad j = 1, 2, \ldots, k-1. \tag{4}$$

The second property is that an optimal piecewise monotonic approximation can be generated by solving a separate monotonic approximation problem on each range $[t_{j-1}, t_j]$. Thus, the components y_i, $i = t_{j-1}, \ldots, t_j$ have the values that minimize the sum of squares of residuals

$$\sum_{i=t_{j-1}}^{t_j} (y_i - \phi_i)^2 \tag{5}$$

subject only to the constraints $y_i \le y_{i+1},\ i = t_{j-1}, \ldots, t_j - 1,$ if j is odd, or subject to the constraints $y_i \ge y_{i+1},\ i = t_{j-1}, \ldots, t_j - 1,$ if j is even. We denote the least value of (5) by $\alpha(t_{j-1}, t_j)$ and $\beta(t_{j-1}, t_j)$ for the increasing and decreasing cases, respectively. These values can be calculated in only $\mathcal{O}(t_j - t_{j-1})$ computer operations. Therefore, if the optimal sequence $\{t_j\colon j = 1, 2, \ldots, k-1\}$ is available, then the sum of squares of residuals of the calculated fit has the value

$$\sum_{i=1}^{n}(y_i - \phi_i)^2 = \alpha(t_0, t_1) + \beta(t_1, t_2) + \alpha(t_3, t_4) + \cdots + \alpha(t_{k-1}, t_k)[\text{ or } \beta(t_{k-1}, t_k)]. \tag{6}$$

We define

$$\gamma(\ell, t) = \sum_{i=1}^{t}(u_i - \phi_i)^2 \tag{7}$$

to be the value of the partial sum by an optimal fit $\{u_i : i = 1, 2, \ldots, t\}$ in $\mathbb{R}^t$ with ℓ monotonic sections. Then, we deduce the first ℓ terms of sum (6) by the following recursive procedure. The calculation begins from $\gamma(1, t) = \alpha(1, t)$, for $t = 1, 2, \ldots, n$, and, as $\ell = 2, 3, \ldots, k$, proceeds by applying the formulae

$$\gamma(\ell, t) = \begin{cases} \min\limits_{1 \le s \le t} \left[\gamma(\ell - 1, s) + \alpha(s, t)\right], & \ell \text{ odd} \\ \min\limits_{1 \le s \le t} \left[\gamma(\ell - 1, s) + \beta(s, t)\right], & \ell \text{ even,} \end{cases} \tag{8}$$

for $t = 1, 2, \ldots, n$ and storing $\tau(\ell, t)$, which is the value of s that minimizes expression (8), for each value of ℓ and t. When the process reaches $\ell = k$ and $t = n$, the value $\tau(k, n)$ is the integer t_{k-1} that occurs in Eq. (8). Hence, with $t_0 = 1$ and $t_k = n$, the optimal values $\{t_j : \ j = 1, 2, \ldots, k-1\}$ are obtained by the backward formula

$$t_{\ell-1} = \tau(\ell, t_\ell), \text{ for } \ell = k, k-1, \ldots, 2. \tag{9}$$

The components of the corresponding optimal approximation are obtained by independent monotonic approximation calculations between successive t_j, as described in the paragraph that includes formula (5).

So far, the value of k is provided by the user. Vassiliou and Demetriou [10, 11] have studied the problem of estimating adequate values of k automatically by testing for data trends due to errors, and, also, by some statistical tests [12].

3 Application to UK Covid-19 Data

This section applies the piecewise monotonic approximation method to Covid-19 data of the UK in order to identify main trends and important peaks. Two datasets are used, which contain the daily Covid-19 new cases and new deaths in the UK, for the period 31 January 2020 through 19 November 2021. The data were downloaded from the official UK government website for data and insights on coronavirus (Covid-19) [9], developed as an open-source service by the UK Health Security Agency.[1] In the following pages, we use the term “turning point” for the value y_{t_j} at the integer variable t_j. We also refer to y_{t_j} as a peak when j is an odd integer, and as a trough when j is even.

3.1 Covid-19 New Cases

The UK Covid-19 new cases datafile contains $n = 659$ pairs of data points $\{(x_i, \phi_i) : i = 1, 2, \ldots, n\}$, where the first coordinate represents the date in year, month and day format, and the second coordinate represents the number of daily Covid-19 new cases. The main characteristics of the data are illustrated in Fig. 1. We can distinguish ranges of small variability, some noticeable peaks that vary in magnitude, and many peaks at lower levels. Not an obvious pattern is apparent.

We seek turning points that have real importance in the analysis of Covid-19 data, but no knowledge of the underlying function $f(x)$ is required. Since a suitable value of k is not initially known, we run the method for a sequence of integers k. Specifically, we fed the data to our software package by trying different values of

[1] https://www.gov.uk/government/organisations/uk-health-security-agency.

Table 1 Left four columns: Turning points in the Covid-19 new cases data by a best fit with $k = 15$ monotonic sections

j	t_j	x_{t_j}	ϕ_{t_j}	$k =$	3	5	7	9	11	13	15
0	1	31-Jan-20	2		×	×	×	×	×	×	×
1	68	07-Apr-20	5486							×	×
2	154	02-Jul-20	6							×	×
3	287	12-Nov-20	33517				×	×	×	×	×
4	304	29-Nov-20	12164				×	×	×	×	×
5	344	08-Jan-21	68192		×	×	×	×	×	×	×
6	458	02-May-21	1674		×	×	×	×	×	×	×
7	534	17-Jul-21	54183			×	×	×	×	×	×
8	551	03-Aug-21	21855			×	×	×	×	×	×
9	574	26-Aug-21	38117								×
10	578	30-Aug-21	26285								×
11	582	03-Sep-21	42355						×	×	×
12	595	16-Sep-21	26619						×	×	×
13	630	21-Oct-21	51719					×	×	×	×
14	647	07-Nov-21	29889					×	×	×	×
15	659	19-Nov-21	44835		×	×	×	×	×	×	×
				[a] $\gamma(k, n) =$	6.40_9	3.77_9	2.87_9	2.03_9	1.61_9	1.22_9	1.12_9
				$D =$	2.14_4	1.34_4	1.13_4	8.55_3	7.65_3	7.65_3	7.65_3

Right seven columns: The turning point positions of the optimal fit for $k \in \{3, 5, \ldots, 15\}$ are indicated by the times sign

[a] subscripts indicate multiplication by 10 raised to the power of the subscript

$k \in \{1, 2, 3, \ldots\}$. Preliminary experimental outcomes suggest that there is no need for best approximations with more than $k = 13$ monotonic sections. In fact, this fit reveals the major peaks of the data and the in-between trends that seem to have significance. Piecewise monotonic fits with only $k = 3, 5, 7$, and 13 monotonic sections are presented.

In Table 1, we summarize our findings. Herein, we present the values and positions of the turning points of each best fit with k monotonic sections to the Covid-19 new cases data for the seven successive odd values of k in the set $\{3, 5, 7, 9, 11, 13, 15\}$. This table is made of two parts of columns, and 16 rows, one row for each turning point of the fit when $k = 15$, and also includes $t_0 = 1$ and $t_{15} = 659$. The left-hand part presents j, t_j, x_{t_j} and ϕ_{t_j}, while we remind that $\phi_{t_j} = y_{t_j}, 1 \leq j \leq n-1$. The right-hand part has seven columns, one for each value of k. The times symbol in each row shows the turning points positions (t_j) of each optimal fit for the corresponding value of k. As an illustration, when $k = 7$, we see that the turning points are at positions 287 (peak), 304 (trough), 344 (peak), 458 (trough), 534 (peak), 551 (trough) as shown by the times signs in the corresponding column labeled "7". The sum of squares of residuals, say $\gamma(k, n)$, and the maximum absolute residual, say $D = \max_{1 \leq i \leq n} |y_i - \phi_i|$, of each fit for a particular value of k can be seen at the bottom of Table 1. For example, when $k = 7$ we have the values $\gamma(k = 7, n = 659) = 2.87 \times 10^9$ and $D = 1.13 \times 10^3$ respectively.

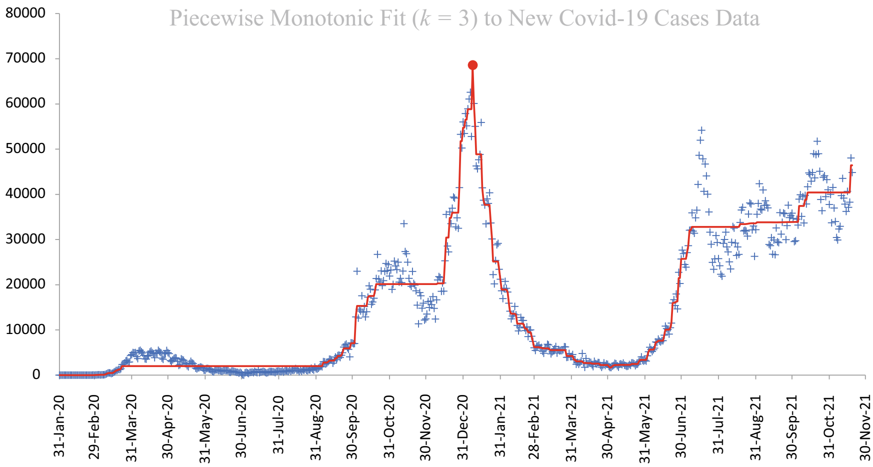

Fig. 1 Detected peak (circle) by a best piecewise monotonic fit with $k = 3$ to the 659 data points (plus signs) of the Covid-19 new cases data during the period 31 January 2020–19 November 2021. The solid line illustrates the best fit. The date and the number of Covid-19 new cases are given in the x-axis and y-axis, respectively

In Fig. 1, we plot the best approximation with $k = 3$ monotonic sections. Here we see the most important peak that the method has identified along the data sequence. In association with Table 1, we see that the objective function and the maximum absolute residual are equal to $\gamma(k = 3, n = 659) = 6.40 \times 10^9$ and $D = 2.14 \times 10^4$ respectively.

Figures 2, 3 and 4 display the results as we apply the method with $k = 5, 7$ and 13, respectively, in order to identify additional turning points of real importance within the data. The best approximation with $k = 5$ obtained four turning points and reduced $\gamma(k, n)$ to 3.77×10^9, as we see in Table 1. Figure 2 displays the data, the fit and the turning points. The new fit maintained the peak of the fit presented in Fig. 1, while two more turning points were detected, whose coordinates can be seen in Table 1.

The best approximation with $k = 7$ obtained two more turning points with indices 287 and 304 respectively, whose coordinates can be seen in Table 1. In return, we see in Fig. 3, which displays the fit with $k = 7$, that the left-hand-side part of the fit that occurs in Fig. 2 is enhanced with one extra peak. Further, $\gamma(k, n)$ is reduced to 2.87×10^9.

We considered also the fit when $k = 13$. Now, six more turning points were added automatically to the fit of Fig. 3, which resulted to three extra peaks, as we see in Fig. 4. Again, $\gamma(k, n)$ is reduced to 1.22×10^9, and the coordinates of the turning points can be seen in Table 1.

A comparison of Figs. 1 through 4 reveals the differences of the corresponding fits to the dataset with respect to the values of k. It is noticeable that when $k = 13$, the method succeeds at identifying a piecewise monotonic approximation which has effectively detected the 12 most important turning points. On the other hand, smaller

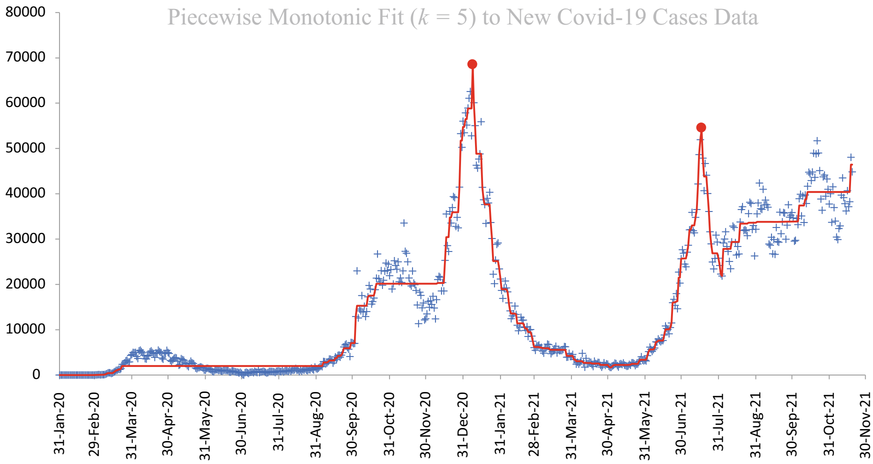

Fig. 2 As in Fig. 1, but $k = 5$

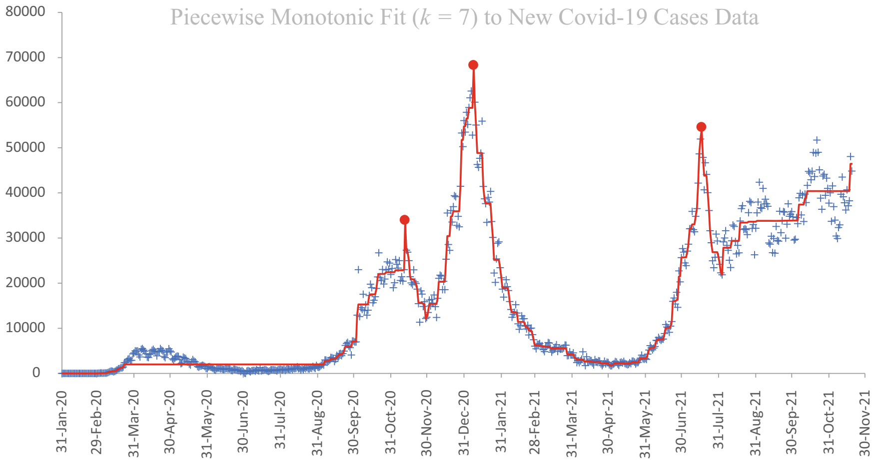

Fig. 3 As in Fig. 1, but $k = 7$

values of k give a smaller number of monotonic sections for the resultant fit, but significant underlying trends in the data are not captured from the corresponding optimal approximations. As the value of k increases, the method continually detects additional monotonic trends in the data, which are not detected in the previous steps.

We see in Table 1 that $\gamma(k, n)$ decreases from 6.40×10^9 down to 1.12×10^9 as we obtain best fits with $k = 3$ up to $k = 15$ monotonic sections. Analogously, the values of the maximum absolute residual are reduced from 2.14×10^4 down to 7.65×10^3 as k increases, which shows that the optimal approximation approaches the data. An important feature of the method is that it detects and adds to the fit turning points in a way that achieves the largest decrease in the sum of squares of residuals. Of note is that as k changes, the most important turning points are

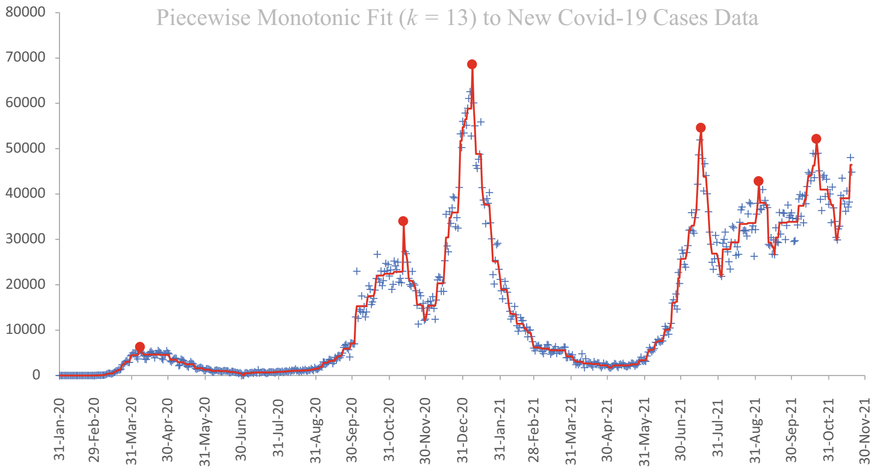

Fig. 4 As in Fig. 1, but $k = 13$

maintained. Further, a comparison of the columns of Table 1 shows the differences in the final fits to the Covid-19 new cases data. It worth noticing that all the turning points of the best approximation with k monotonic sections are turning points of the best approximation with $k + 2$ monotonic sections. We note, though, that this is due to the particular data, and not a property of the optimal approximations whose number of monotonic sections differs by two.

3.2 Covid-19 New Deaths

We estimate peaks of the Covid-19 new deaths data for the UK. The data consist of 659 daily number of deaths from Covid-19 disease from January 31, 2020 through November 19, 2021. The datafile format is the same as described in Sect. 3.1, except that the second column keeps the number of deaths due to Covid-19. We capture the main features of these data by looking at Fig. 5. Indeed, we can see that the data include large deviations, as well as two distinguishable peaks with sharp increases.

With these data, we run our computer program for $k = 3$, 5 and 7. For the sake of example, the best approximation when $k = 5$ detects four turning points which have indices 71 (peak), 213 (trough), 356 (peak) and 488 (trough). Figure 5 also displays the resultant fit and the peaks.

Table 2 presents some results analogous to Table 1 for $k = 3, 5, 7$. Here, the sum of squares of residuals is reduced from 2.47×10^7 down to 6.71×10^6 as k increased from $k = 3$ to $k = 7$ by step 2. The maximum absolute residual decreased from 1.02×10^3 down to 5.43×10^2 as k increased from $k = 3$ to $k = 7$. Thus, all the conclusions of Sect. 3.1 hold for the Covid-19 deaths data examined in this section.

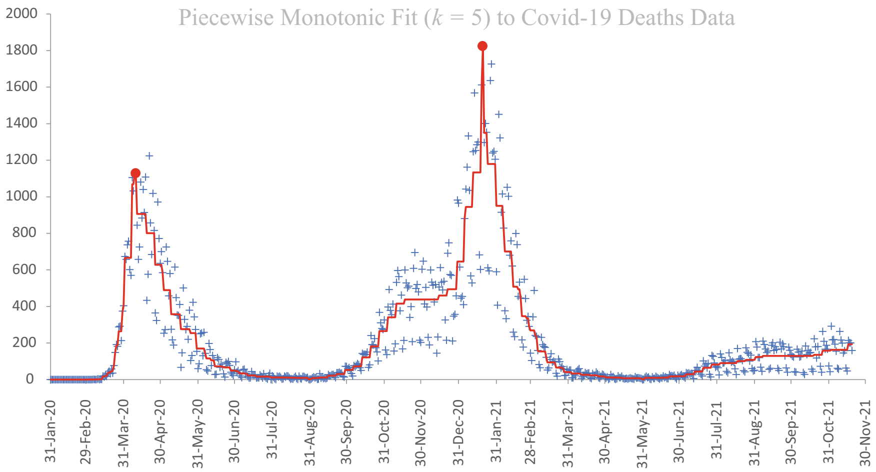

Fig. 5 Detected peaks (circles) by a best piecewise monotonic fit with $k = 5$ to 659 data points (plus signs) of Covid-19 new deaths data during the period January 31, 2020–November 19, 2021. The solid line illustrates the best fit. The date and the number of deaths are given in the x-axis and y-axis, respectively

Table 2 Left four columns: Turning points in the Covid-19 deaths data by a best fit with $k = 7$ monotonic sections

j	t_j	x_{t_j}	ϕ_{t_j}	$k =$	3	5	7
0	1	31-Jan-20	0		×	×	×
1	71	10-Apr-20	1123			×	×
2	213	30-Aug-20	1			×	×
3	356	20-Jan-21	1826		×	×	×
4	361	25-Jan-21	594				×
5	363	27-Jan-21	1726				×
6	488	01-Jun-21	0		×	×	×
7	659	19-Nov-21	159		×	×	×
				$\gamma(k, n) =$	2.47_7	7.89_6	6.71_6
				$D =$	1.02_3	5.86_2	5.43_2

Right three columns: The turning point positions of the optimal fit for $k \in \{3, 5, 7\}$ are indicated by the times sign

4 Analysis Through the Timeline of the Pandemic

The results presented in Tables 1 and 2 in Sect. 3 are used in conjunction with a timeline of the Covid-19 pandemic in the UK, in order to analyze the progress of the pandemic in the UK throughout the studied period and the performance of the piecewise monotonic approximation in doing so accurately, identifying reasons of the existence of the peaks and troughs that the algorithm detects. In order to be consistent with the results of the previous section, we shall analyze the cases and deaths data separately.

4.1 New Covid-19 Cases

The new cases data, as depicted in Figs. 1 through 4, suggest the existence of three major waves of Covid-19 in the UK, throughout the studied period. Considering the earliest measurement as the start of a wave, the first wave—by and large the weakest of the three in terms of cases-peaks at $\phi_{68} = 5486$ cases on the 7th of April 2020—is detected by the algorithm for values of $k \geq 13$, in accordance to the output of Table 1. It bottoms out on the 2nd of July 2020, at $\phi_{154} = 6$ cases, after which the beginnings of the second wave slowly build up. The difference in scale of measures between this wave and the ones that followed it is such that the algorithm officially acknowledges its existence only after every major peak in all subsequent waves has been identified, as far as the daily cases data are concerned (in the results presented, this occurs for $k \geq 13$); this, in itself, is a testament to its smaller scale in terms of cases, as its point of recognition borders on having the algorithm overfit the data. The first wave also saw the first national Covid-19 lockdown in England enforced on the 26th of March, 2020. The period following its enforcement was characterized by a stabilization of daily cases, which lead to their downward trend from April onward. Its lockdown measures lasted well into July, which is also consistent with the very low readings of new cases in this period (the 1st and 2nd of July 2020 bear the smallest figures after the wave abated); after this period, lockdown measures were slowly lifted.

The second wave comes into force in September 2020 and is uniquely identified by a double-peak. The first peak is detected by the algorithm on November 12, 2020, at $\phi_{287} = 33517$ cases; it is to be noted that the number of cases on that day was something of an outlier in the data as unusually high compared to other days in the vicinity of November 12th, but, nonetheless, the data did peak in that area. This peak appears a week after the second Covid-19 lockdown in England is enforced, which supports the descent in cases afterward. The second peak, dominant in the second wave, was detected on January 8, 2021, at $\phi_{344} = 68192$ cases, following the holiday season; just two days prior, the third national lockdown in England had been enforced, which, extending over time as it did, supported the rapid descent in cases and led to the end of the second wave, dropping down to the trough $\phi_{458} = 1674$ cases on May 2, 2021.

Measures to contain the spread of contagion—at least to some extent—were taken from the start of the wave (in contrast to the first wave, which had few measures taken, aside from the lockdown after it went into full force); most notable are the introduction of the “Rule of Six” on the 14th of September 2020, which limited social gatherings to groups of at most six people, and the deployment of a National Health Service Covid-19 tracing app in England and Wales, which was geared toward tracing Covid-19 contagion and advising users to self-isolate accordingly; hundreds of thousands of self-isolation notices were sent over the course of the second wave, likely preventing a more intense surge in cases than what would be the case otherwise.[2] Still, the increase in cases did prompt the imposition of a second national lockdown in England on the 5th of November, 2020, which lasted until the 2nd of December and was met by a decrease in cases, supplanted by self-isolation; of note is that the 8th of December, less than a week later, marked the start of vaccination efforts. After the second lockdown was lifted, the second part of the wave came into effect with the holiday period, rapidly coming to a peak on the 8th of January 2021, with $\phi_{344} = 68192$ cases as previously mentioned; in efforts to stifle the flow of infections, a third national lockdown in England was enforced just two days prior, leading to a rapid descent in cases after the January peak.

There are several characteristics surrounding the two peaks of ϕ_{287} and ϕ_{344} that categorize them both as being in the second wave. For one, the peaks are temporally quite proximal to one another, with a timespan of little over a month between the two. One can readily compare this to the seven-month gap from the peak of the first wave at ϕ_{68} and the six-month gap to the next encountered peak at ϕ_{534}. The latter distance is also supplanted by a deep trough on May 2, 2021 (of $\phi_{458} = 1674$ cases). In addition, the trough detected between the November 12th and January 8th peaks is markedly higher compared to the troughs on either end of the wave, standing at $\phi_{304} = 12164$ cases on November 29th; certainly lower than the two peaks surrounding it, but more than twice the peak of the first wave, as detected by the algorithm. Overall, there is a swift transition from the November descent to the January spike. In contrast, the deep drop in cases after the January spike, brought about by the third lockdown, was maintained well into July 2021, with very gradual liftings of restrictions.[3]

The third wave comes after the lull following the third national lockdown in the second wave. It shows signs of beginning after the trough of $\phi_{458} = 1674$ cases on May 2, 2021 and, at the time of the most recent observation in the studied period, that of November 19, 2021, is still in effect. This wave in particular exhibits a multitude of peaks of varying heights and is of interest, not only over its unusually variant data, but also because the data are such that a sequential increase in k will not necessarily have the algorithm detect its peaks in sequence, as well. It is to be noted that all peaks and troughs after May 2, 2021 can be considered part of the one third wave for the same reasons that the peaks of the second wave are categorized into that wave—namely, temporal proximity among one another and a great distance from any previous peaks.

In regard to the unusual variance of data in this third wave, it is, at least in part, due to the fact that the data for the United Kingdom are aggregates of the data of its four component countries (England, Wales, Scotland and Northern Ireland), meaning that events and behaviors in the data of one of the four may well disrupt the aggregate. For instance, in the period of August 2021, Scotland saw a rampant increase in

[2] Granted, the great volume of self-isolations had in itself other adverse impacts on the workings of the economy, such as the restocking shortages as a result of the "pingdemic" in the third wave.

[3] Of note, is that the final lifting of restrictions was initially delayed due to the advent of the third wave, heralded by the Delta Covid-19 variant in the UK.

cases following the beginning of its school year and lifting of most Covid-related restrictions,[4] which did cause a disruption in the UK data on the whole.

In regard to detecting the peaks of the third wave, the results of the algorithm can be readily observed through Table 1: the first and highest peak of the wave on July 17, 2021, which is also the second-highest peak in all the data at $\phi_{531} = 54183$ cases, is the first of this wave to be detected by the algorithm; in fact, for $k \geq 5$, detection from the algorithm will always grant it precedence over the other peaks in the same wave. However, increasing k to 7 will detect the peak of $\phi_{287} = 33517$ cases on November 12, 2020 (the first peak of the second wave) over the peak of $\phi_{630} = 51719$ cases on October 21st 2021 (the third peak of the third wave, detected for $k \geq 9$, as seen in Table 1 and observed by comparing Figs. 2 and 3). The second peak of the wave, $\phi_{582} = 42355$ cases on September 3, 2021, is the last peak of the third wave to be identified, for $k \geq 11$. In general, aside from the abnormally sudden spike in July, the third wave is clearly in effect, with a steadily increasing trend in cases.

In general, if one were not to mind some misapproximation of the first wave, relatively minor as it may be, the results of the algorithm for $k = 3, 5, 7, 9$ can provide decent results in terms of both approximation and trend charting. The results for $k = 7$ in particular encompass most major points of the second and third waves (all peaks and troughs of the second wave, the July spike and the general upward trend of the third wave). Values of k beyond 13, though accurately approximating the first wave, grow increasingly prone to overfitting for only minor benefits in terms of residuals (as can be seen in Table 1, for $k \geq 11$, the maximum absolute residual D remains unchanged at 7.65×10^3 while the sum of squared residuals sees relatively minimal changes from 1.66×10^9, to 1.12×10^9); in fact, for $k = 15$, the added turning points on the 26th and 30th of August 2021 can be considered extraneous, when compared to the general trend of the surrounding data in that subperiod. This suggests that $k = 13$ is the point where one should stop seeking to increase k, in terms of these data.

4.2 New Covid-19 Deaths

The three Covid-19 waves that appear in the new cases data are also reflected in the data for daily Covid-19-related deaths, with three apparent waves of deaths. Notably, there is a time lag between the surges in cases and their associated surges in deaths, usually attributed to the time period over which a case might become a fatality. Since not all who contract the virus expire, there is a significant difference in scale between the measures of cases and deaths, as can readily be seen by comparing the respective Tables 1 and 2; the peaks of the deaths reach just below 2000, while cases soar as

[4] Coronavirus (COVID-19) update: First Minister's statement—24 August 2021, https://www.gov.scot/publications/coronavirus-covid-19-update-first-ministers-statement-24-August-2021/.

high as above 68000. The best approximation with $k = 5$ monotonic sections to these data is seen on Fig. 5. The suitability of the value of k is explained below.

The first wave of deaths is asserted by the algorithm to peak on April 10, 2020, at $\phi_{71} = 1123$ deaths on that day, leading to a trough of $\phi_{213} = 1$ death on August 30, 2020. It is worth pointing out, as seen by comparing the cases figures with Fig. 5, that the first wave, for its comparatively diminutive cases surge, had among the highest deaths-to-cases ratio across all three waves in the studied period; this makes sense, as it was the first time the medical system had to deal with Covid-19.

From August 30, 2020 onward, the second wave comes into effect; the algorithm recognizes a single peak for this surge of deaths, of $\phi_{356} = 1826$ deaths on January 20, 2021. It is worth noting that the algorithm does not detect a death peak corresponding to the cases peak of November 12, 2020; this does make sense, as there is not nearly as pronounced a descent in deaths before the peak in January as there was in cases, which works against the algorithm suggesting the existence of a trough between the peaks. Following the expected descent in cases and deaths, the second wave bottoms out at the trough on June 1, 2021, at $\phi_{488} = 0$ deaths, leading into the beginnings of the third wave.

It is very much worth noting how much milder this wave of deaths is compared to its predecessors; while the first Covid-19 wave was severely outclassed in terms of cases, its fatalities were among the highest in a short period of time—the January 2021 spike of $\phi_{363} = 1826$ notwithstanding. On the other hand, the third wave has by far the lowest number of deaths per day among all waves in this studied period. This can, if partly, be explained by the fact that vaccination efforts are already in full swing by this point, which the previous two waves lacked. In addition to knowledge and experience accumulated by combating the pandemic over the year or so prior, there are several factors that can explain why this wave of deaths shows lower daily measures, even in the face of the spike in cases on July 2021 ($\phi_{531} = 54183$ in the daily cases data). That said, however, even low daily measures add up over extended periods of time, so it still is a serious measure to consider, especially considering that the data here, too, display a steady, if slow, rising trend.

Unlike the case of the new cases data, a value of $k = 5$ readily seems to be the most appropriate to approximate this data. With four turning points, all major waves are approximated and their trends detected. Even if this were not the case, any value of k that is higher than five becomes prone to gross overfitting of the data; as seen in Table 2, for $k = 7$, the algorithm asserts peaks and troughs exist on the rapid descent after the January 2021 spike. A “trough” of $\phi_{361} = 594$ deaths on January 25, 2021, just 5 days after the original spike on January 20th, and a “peak” of $\phi_{363} = 1726$ deaths on January 27th, just two days after the aforementioned “trough”; these results are readily regarded as erroneous and a prime example of overfitting, given that the trend they suggest for that seven-day span stands against the clearly descending trend suggested before and after them. This is an interesting point to note; despite being intrinsically linked, the datasets of deaths and cases provided quite different results in terms of the choice in k, given their difference in structure.

5 Conclusions

Piecewise monotonic approximation as a data smoothing technique can have many applications. In this work, we applied it to Covid-19 new cases and deaths data of the UK in order to estimate important turning points and major trends. These data exhibit large variations. Although the minimization calculation may have a very large number of local minima, our algorithm obtains a global solution routinely by considering all needed combinations of turning points. For example, given that $n = 659$, when $k = 13$, the number of trials would be of magnitude n^{12}, but we obtain a solution in about 0.001 s on a common pc.

The method constructs an approximation to the data that obtains the piecewise monotonicity property and, generally, can accommodate undulation of the data by choosing a suitable k. By increasing k, piecewise monotonic approximation reveals the most important turning points in a sequence. It also makes the sum of the squares of the residuals smaller and reduces the maximum absolute residual, while in practice it maintains the most important turning points provided by former values of k. A considerable advantage of piecewise monotonic approximation in peak estimation is that the presence of a peak in the data does not introduce any perturbations into the approximation.

As mentioned in Sect. 1, the application and analysis of the piecewise monotonic method to Covid-19 data are attempted for the first time. Applying the method to the Covid-19 data of the UK yielded significant results. For one, the fittings to the daily data provided by the algorithm proved to quite adequately capture the trends of the three waves of the pandemic in the studied period, in addition to approximating the data. Furthermore, the differences present in the structure of the data between the datasets of daily cases and daily deaths provided ample opportunity to ascertain the effects of various values of k on the resulting fits. One such effect would be the trade-off between detecting all Covid-19 waves in the cases data and overfitting said data. Another would be the difference in the optimal values of k between the cases and deaths data in spite of their intrinsic link. Additionally, viewing the data through the lens of various significant events pertaining to the progress of the pandemic in the UK provided us with insights regarding the driving forces behind the peaks, troughs and trends that the algorithm detected.

One may well combine certain features of the piecewise monotonic approximation method with other techniques, if there exists an opportunity for further analyses in estimating turning points and trends, and in developing a relevant ongoing system with predictive accuracy. The numerical results of this paper and the insights into Covid-19 drew our attention to some interesting questions on the piecewise monotonic approximation method and its application to the pandemic data that deserve further study.

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