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Least squares monotonic unimodal approximations to successively updated data and an application to a Covid-19 outbreak

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ABSTRACT

Approximations to noisy data are constructed to obtain unimodality of successively updated data. Precisely, a method is developed that calculates least squares monotonic increasing - decreasing approximations to n data, for successive values of n. Having fixed n, the statement of the increasing-–decreasing constraints in terms of first differences of the data subject to one sign change gives rise to a discrete search calculation, because the position of the sign change is also an unknown of the optimization problem. Although this problem may have many local minima for each value of n, the method is made quite efficient by taking advantage of two properties of the optimization calculation. The first property, for fixed n, provides necessary and sufficient conditions that reduce the optimal selection of the position to solving two sequences of monotonic approximation problems to subranges of data in the least complexity $\mathcal{O}(n)$. The sufficiency conditions is a new result. The second property shows that the optimal position increases as n increases. Hence, if this position of a fit to the first $n-1$ data is available, then in order to calculate the required position of a fit to the first n data there is no need to consider data that are before the current position. A Fortran program has been written and some timings of numerical results are given for up to $n = 30{,}000$ data and various levels of noise. They seem to be proportional to n. The method is applied to data of daily Covid-19 deaths of the United Kingdom for a period that exhibits a major outbreak, and obtains insights of the evolution of the process as new data enter the calculation. The results reveal certain features of the calculation that may be helpful to supporting policy making, when used in modelling epidemic processes. Further, a procedure is proposed where our method initializes the nonlinear least squares calculation of Richards epidemiological model, and the numerical results confirm some useful advantages over the plain use of this model. One benefit of our method for unimodal smoothing of successive data and estimation of the associated turning points is that it provides a property that appears in a wide range of processes from biology, economics, finance and social sciences, for instance.

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